A General Equivalent Circuit Model for Piezoelectric Generators

NIELL G. ELVIN[1,*] AND ALEX A. ELVIN[2]

[1]*Department of Civil and Environmental Engineering, Michigan State University Engineering Building 3573, East Lansing, MI 48824, USA*

[2]*Department of Civil and Environmental Engineering, University of the Witwatersrand Johannesburg, South Africa*

ABSTRACT: This article presents an equivalent circuit model for a piezoelectric generator which can include any number of vibrational modes. First the electromechanical equations are formulated using an assumed mode (for example, the Rayleigh-Ritz method), the mechanical equations are then decoupled by the standard eigenvector approach. A set of single degree of freedom equations are thus produced. The electromechanical coupling terms are modeled in the equivalent circuit using an ideal transformer, or a set of current-and voltage-dependent sources. To validate the equivalent circuit model, the results show excellent agreement with published analytical solutions for the first three vibration modes of a cantilever unimorph generator. The main advantage of the new method is that it can be used to simulate any circuit topology, for which there is no analytical solution, using a standard electronic simulation program. To demonstrate this, the analysis and design of a more complicated diode bridge circuit is presented.

Key Words: vibrational energy harvesting, multiple degrees of freedom, SPICE.

INTRODUCTION

THE proliferation of low-power wireless sensors has led to increased interest in converting ambient mechanical vibration into electrical energy. Such energy harvesting provides a renewable power source that does not require periodic replacement or recharging of batteries. The proposed use of electrical harvesters has included the monitoring of both machines and structures, and is of wide interest in the electrical, mechanical, and civil engineering communities. Piezoelectric energy generation has been extensively studied as one of the relatively inexpensive and high-power density alternative in vibrational energy scavenging. One of the major research challenges in the field of piezoelectric generators includes the ability of accurately modeling and designing these coupled electromechanical systems. Several researchers have proposed various models ranging from simple single degree of freedom (SDOF) models (Roundy and Wright, 2004; du Toit et al., 2005; Ajitsaria et al., 2007) to more complicated assumed mode and distributed parameter models (Sodano et al., 2004; Erturk and Inman, 2008). Erturk and Inman (2008) have recently found the closed formed solution of the cantilever piezoelectric generator under a resistive load. The SDOF models cannot accurately capture the higher vibration modes and are inaccurate when the modes are closely spaced. Although considering the higher modes producing more accurate solutions, they have been included only with a resistive load acting as the electrical circuit.

It is beyond the scope of this article to give an extensive overview of piezoelectric generator models which has been done by others (Roundy and Wright, 2004; Sodano et al., 2004).

In reality, most circuits that are attached to the piezoelectric harvester are more complex than a single resistor (as has been modeled extensively in the past), and might include nonlinear elements. The accurate modeling of these circuits would require electronic circuit simulators such as SPICE. Some equivalent 1-port (Roundy and Wright, 2004) and 5-port (Cho et al., 2000) circuits have been proposed for modeling particular piezoelectric behavior, but a generalized equivalent circuit model that can capture *any number* of piezoelectric modes and can be easily incorporated into a circuit simulator has not been proposed to date. In this article we present a derivation of an equivalent electric circuit model for a cantilever piezoelectric

*Author to whom correspondence should be addressed. Email: elvin@msu.edu
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generator excited by transverse base motion. The circuit model is easily extendable to capture any number of vibrational modes. Furthermore, the equivalent circuit for generators with different boundary condition, material properties, and geometry can be derived in an analogous way. The results from a derived equivalent model are compared with the analytical solution of Erturk and Inman (2008) for the first three vibrational modes of cantilever unimorph generator.

DERIVATION OF EQUATIONS

The coupled piezoelectric equations are derived from Hamilton's principle following a similar procedure described by Sodano et al. (2004), Hagood et al. (1990) and Humar (1989).

If the piezoelectric cantilever subjected to base excitation (Figure 1) is assumed to behave as an Euler–Bernoulli beam, the longitudinal strains (S_{11}) are given by:

$$S_{11}(x,z,t) = -z\frac{\partial^2 u(x,t)}{\partial x^2} = -zu_{xx} \tag{1a}$$

Where u is the vertical deflection of the beam, and z is the perpendicular distance from the neutral axis.

The electrical potential (Ψ) is assumed to vary linearly through the piezoelectric thickness, and is given by:

$$E(z,t) = \frac{\partial \Psi(z,t)}{\partial z} = \Psi_z = \begin{cases} \frac{V}{h_p} \text{ on the outer electrode surface} \\ 0 \text{ on the inner electrode surface} \end{cases} \tag{1b}$$

Where h_p is the thickness of the piezoelectric layer.

The displacement (u) and electrical potential (Ψ) can be represented in modal and field coordinates as:

$$u(x,t) = \phi_1(x)r_1(t) + \phi_2(x)r_2(t) + \cdots + \phi_m(x)r_m(t) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{r}$$
$$\Psi(x,t) = \psi_1(z)v_1(t) + \psi_2(z)v_2(t) + \cdots + \psi_n(z)v_n(t) = \mathbf{\Psi}^{\mathrm{T}}\mathbf{v} \tag{2}$$

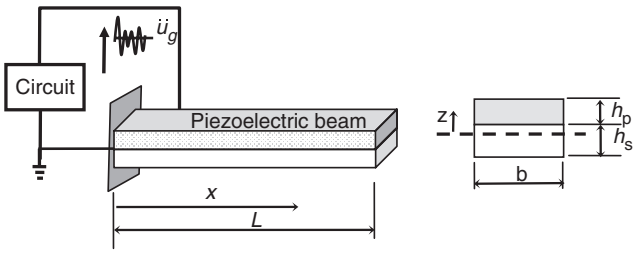

Figure 1. *Schematic depiction of a cantilever beam, unimorph piezoelectric vibration scavenger of length (L), width (b), and height ($h = h_p + h_s$). The dotted line represents the neutral axis.*

where $\mathbf{\Phi}$ is a vector of assumed mode shapes for the structure, $\mathbf{r}$ is a temporal displacement vector, Ψ is the assumed field vector, and $\mathbf{v}$ is the piezoelectric voltage. This leads to a set of equations of the form:

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{D}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} - \mathbf{\Theta}\mathbf{v} = -\mathbf{M}^*\ddot{u}_g \tag{3a}$$

$$\mathbf{\Theta}^{\mathrm{T}}\dot{\mathbf{r}} + \mathbf{C}\mathbf{v} = \mathbf{I} \tag{3b}$$

where $\mathbf{M}$, $\mathbf{D}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices, respectively. $\mathbf{\Theta}$ is the piezoelectric coupling vector, $\mathbf{C}$ is the capacitance vector, $\mathbf{I}$ is the vector of piezoelectric currents, $\mathbf{M}^*$ is the effective forcing vector, and $\ddot{u}_g$ is the applied base acceleration. A dot above the variable represents a derivative with respect to time. The entries in the mass, stiffness, and proportional damping matrices with indices i and j are:

$$k_{ij} = \int_{\mathrm{Vol}} Y^{\mathrm{E}} z^2 \phi_i''(x)\,\phi_j''(x)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$
$$m_{ij} = \int_0^L \bar{m}\phi_i(x)\,\phi_j(x)\,\mathrm{d}x + M_{\mathrm{tip}}\,\phi_i(L)\phi_j(L) \tag{4}$$
$$d_{ij} = \alpha_0 m_{ij} + \alpha_1 k_{ij}$$

where Y^{E} is the short circuit modulus of elasticity, $\bar{m}$ is the mass per unit length of the beam, M_{tip} is the tip mass (if present), and α_0 and α_1 are experimentally derived damping factors. A superscript dash represents derivative with respect to length x. The entries in the effective forcing vector are given by:

$$m_i^* = \int_0^L \bar{m}\phi_i(x)\,\mathrm{d}x + M_{\mathrm{tip}}\,\phi_i(L) \tag{5a}$$

The piezoelectric matrices are:

$$\theta_{ij} = \int_{\mathrm{Vol}} z\phi_i''(x)e_{31}\psi_j(x)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \quad \text{and}$$
$$c_{ij} = \int_{\mathrm{Vol}} \varepsilon\psi_i(z)\,\psi_j(z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \tag{5b}$$

where e_{31} is electromechanical coupling coefficient, $\Psi(z)$ is the electric field, and ε is the electrical permittivity.

In the Rayleigh–Ritz approximation, the assumed displacement modes can be a set of any functions that satisfy the geometric (essential) boundary conditions. For example, for the piezoelectric cantilever beam, the assumed mode shape could be of the form:

$$\phi_i(x) = 1 - \cos\left(\frac{(2i-1)\pi x}{2L}\right) \tag{6}$$

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The electrical field mode shape is often assumed constant over the thickness of the piezoelectric:

$$\psi_j(z) = \frac{1}{h_p} \tag{7}$$

Where h_p is the thickness of the piezoelectric material.

Depending on the assumed mode shape the **K**, **M**, **D**, **M***, and **Θ** matrices can be fully populated. For example, assuming a unimorph piezoelectric geometry with no tip mass, and the first three modes given by Equation (6), these matrices become:

$$\mathbf{M} = \bar{m}L\begin{bmatrix} \frac{3}{2}-\frac{4}{\pi} & 1-\frac{4}{3\pi} & 1-\frac{12}{5\pi} \\ 1-\frac{4}{3\pi} & \frac{3}{2}+\frac{4}{3\pi} & 1+\frac{4}{15\pi} \\ 1-\frac{12}{5\pi} & 1+\frac{4}{15\pi} & \frac{3}{2}-\frac{4}{5\pi} \end{bmatrix}$$

$$\mathbf{K} = \frac{1}{32}\frac{\pi^4 Y I_{xx}}{L^3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 625 \end{bmatrix} \tag{8}$$

$$\mathbf{D} = \alpha_0\mathbf{M} + \alpha_1\mathbf{K}$$

$$\mathbf{\Theta}^{\mathrm{T}} = \frac{b\pi e_{31}}{4L}(2h_{pa} - h_p)\begin{bmatrix} 1 & -3 & 5 \end{bmatrix}$$

$$C = \frac{bL\varepsilon}{h_p}$$

$$\mathbf{M}^{*\mathrm{T}} = \bar{m}L\begin{bmatrix} 1-\frac{2}{\pi} & 1+\frac{2}{5\pi} & 1-\frac{2}{5\pi} \end{bmatrix}$$

Where I_{xx} is the second moment of area, and h_{pa} is the distance from the neutral axis to the top of the piezo.

The equivalent circuit of Equation (8) is not trivial since the coupling (off diagonal terms) in the matrices leads to non-intuitive circuit layouts and components. A method of overcoming this problem is to diagonalize the matrices by solving the $\mathbf{M}\ddot{\mathbf{r}} + \mathbf{K}\mathbf{r} = \mathbf{0}$ eigenvalue problem, which leads to a solution $\mathbf{r} = \mathbf{\Phi}\mathbf{q}$ where **Φ** is the associated matrix of eigenvectors. Substituting this solution into Equation (3) and premultiplying Equation (3a) by $\mathbf{\Phi}^{\mathrm{T}}$ gives:

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}\ddot{\mathbf{q}} + \mathbf{\Phi}^{\mathrm{T}}\mathbf{D}\mathbf{\Phi}\dot{\mathbf{q}} + \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}\mathbf{q} - \mathbf{\Phi}^{\mathrm{T}}\mathbf{\Theta}\mathbf{v} = -\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}^{*}\ddot{u}_g$$

$$\mathbf{\Theta}^{\mathrm{T}}\mathbf{\Phi}\dot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{v}} = I \tag{9}$$

The orthoganility condition of **Φ** insures that the $\mathbf{M} = \mathbf{\Phi}^{\mathbf{T}}\mathbf{M}$, $\mathcal{D} = \mathbf{\Phi}^{\mathbf{T}}\mathbf{D}$, and $\mathcal{K} = \mathbf{\Phi}^{\mathbf{T}}\mathbf{K}$ are all diagonal matrices. It should be noted that the damping matrix is specifically chosen to be mass and stiffness proportional [Equation (8)] so that it can be diagnolized in this fashion. The set of decoupled equations can then be readily converted into an equivalent circuit.

THE EQUIVALENT CIRCUIT

The derivation of the SDOF equivalent piezoelectric circuit has been presented by Roundy and Wright (2004).

In this model, the single equivalent mass, stiffness, and damping is represented by an inductor, capacitor, and resistor. The electromechanical coupling is represented by an ideal transformer (Figure 2).

The current in the mechanical loop is given by $\dot{r}_1$. Using Kirchoff's voltage law:

$$\mathrm{M}_{11}\ddot{r}_1 + \mathrm{D}_{11}\dot{r} + \mathrm{K}_{11}r + v_{ab} = -\mathrm{M}_1^{*}\ddot{u}_g \tag{10}$$

and $v_{ab} = \mathbf{\Theta}_1 v_{cd} = -\mathbf{\Theta}_1 v$ where v is the voltage in the piezoelectric. Note that this equation is identical to Equation 3(a).

Equations 3(b) can be derived by considering the current (i_c) flowing into the piezoelectric capacitor (C). The current is given by Kirchoff's current law as:

$$i_c = i - I \text{ with } i = -\dot{r}_1\mathbf{\Theta}_1 \quad \text{and} \quad \mathbf{i}_c = \mathbf{C}\dot{\mathbf{v}} \tag{11}$$

Consider now a piezoelectric beam in which only the first two modes are of interest. The equations are first decoupled into a set of SDOF systems in the modal coordinate space. These SDOF systems can then be represented by the equivalent circuit as shown in Figure 3. Note its similarity to the one-mode equivalent circuit given in Figure 2. In the modal coordinate system the loop currents are $\dot{q}_i$ and not $\dot{r}_i$.

By analogy the circuit shown in Figure 3 can be extended to include any number of mechanical modes. However, the ideal transformer element (shown in the dashed block in Figure 3) for the higher mode equivalent circuit is a nonstandard electrical element which might not be available in circuit-simulation programs. To solve this problem, the two-mode ideal transformer can also be represented by a coupled set of voltage- and current-dependent sources as shown in Figure 4.

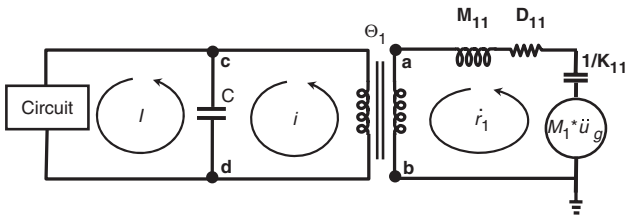

Figure 2. *The equivalent circuit for a single degree of freedom piezo-generator.*

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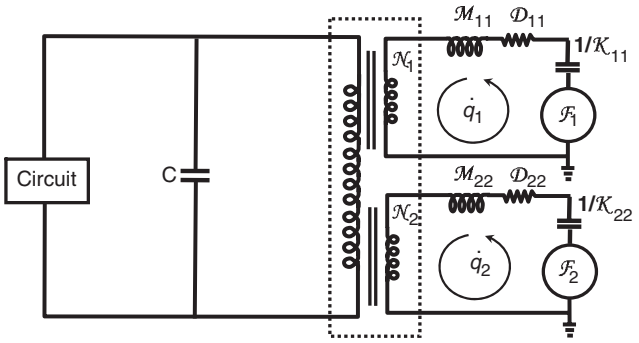

Figure 3. *The equivalent circuit for a two degree of freedom piezo-generator. The symbols* $\dot{q}_2$, $\mathcal{N}_i = (\Phi^T\boldsymbol{\Theta})_i$, *and* $\mathcal{F}_i = (\Phi^T\mathbf{M}^*\mathbf{u}_g)_i$ *represent the velocity, piezoelectric coupling, and force in the modal co-ordinate system.*

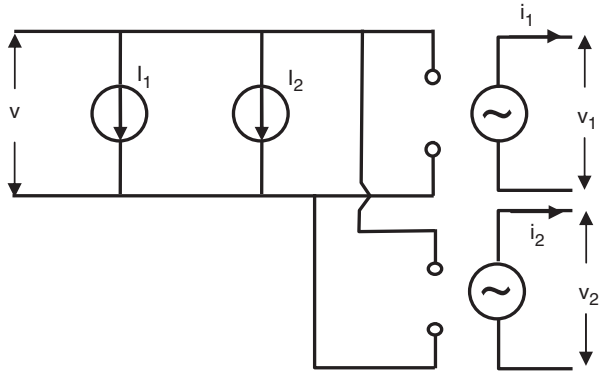

Figure 4. *Circuit representation of a 2-mode ideal transformer. Here the dependent voltage sources (*v_1 *and* v_2*) are given by* $v_1 = \mathcal{N}_1 v$ *and* $v_2 = \mathcal{N}_2 v$*. The dependent current sources (*I_1 *and* I_2*) are given by* $I_1 = \mathcal{N}_1 i_1$ *and* $I_2 = \mathcal{N}_2 i_2$ *.*

The ideal transformer can be easily extended to include more modes by adding more current- and voltage-dependent sources.

MODEL VALIDATION

The equivalent circuit model is validated by comparing the results to the analytical solution of Erturk and Inman (2008). The first three modes of a unimorph cantilever beam are compared for: (a) the open circuit case (see Figure 1 in which the element 'circuit' is not included), and (b) a purely resistive load (the element 'circuit' is a single resistor). The mode shapes in the equivalent circuit are given by Equation (6). The model and material properties are given in Table 1.

A comparison of the first three modes of vibration of the equivalent circuit model and the analytical solution is given in Table 2. For the frequency range of 0–1000 Hz, both three ($i = 1$ to 3) and four terms ($i = 1$ to 4) in Equation (6) are considered in the equivalent circuit model. The model gives excellent agreement with the analytical solution for the lower

Table 1. Material and geometric properties of a unimorph cantilever beam.

Property	Symbol	Value
Elastic modulus piezoelectric	Y_p	66 GPa
Elastic modulus substrate	Y_s	100 GPa
Density piezoelectric	ρ_p	7800 kg/m^3
Density substrate	ρ_s	7165 kg/m^3
Damping constants	α_0	4.894
(1 and 1.3% of critical damping)	α_1	1.2349e-5
Piezoelectric coupling	d_{31}	190e-12 m/V
Electrical permittivity	ε	1.593e-8 F/m
Beam length	L	100 mm
Beam width	b	10 mm
Piezoelectric thickness	h_p	0.4 mm
Substrate thickness	h_s	0.5 mm

Table 2. Comparison of open circuit and closed circuit resonant frequencies of equivalent circuit model and analytical solution (Erturk and Inman, 2008).

Mode	Analytical Resonant Frequency (Hz)	3 Mode Equivalent Circuit Resonant Frequency (Hz)	4 Mode Equivalent Circuit Resonant Frequency (Hz)
1 Open circuit	48.8	48.86	48.83
1 Closed circuit	47.8	47.87	47.83
2 Open circuit	301.5	303.90	303.43
2 Closed circuit	299.6	301.90	301.36
3 Open circuit	839.2	922.78	853.60
3 Closed circuit	838.2	916.51	851.56

resonant frequencies (<0.73% difference). The agreement is worse for the third resonant frequency if only three terms are included; the agreement is much better (within 1%) if four modes are included.

Figure 5 shows a comparison of the computed voltage transfer function [three modes are considered in Equation (6)] with the analytical solution for two different load resistors. Figure 6 shows the tip deflection transfer functions of the cantilever beam [both three and four modes are considered in Equation (6)]. It should be noted that the output from the equivalent circuit model is in terms of modal velocities ($\dot{\mathbf{q}}$) and first needs to be transformed into the basis vector coordinate system as follows: $\dot{\mathbf{r}} = \boldsymbol{\Phi}\dot{\mathbf{q}}$. The tip velocity is given by:

$$\dot{u}(x = L, t) = \phi_1(L)\dot{r}_1(t) + \phi_2(L)\dot{r}_2(t) + \phi_3(L)\dot{r}_3(t) \quad (12)$$

The tip velocity is then integrated to obtain the tip displacement.

When four modes are included in the equivalent circuit model, the solution gives better approximations of both the natural frequencies (Table 1) and the frequency response (Figure 6). A similar improvement is also seen in the voltage-frequency response. It is important to note that further increasing the number of assumed modes, improves the model's accuracy.

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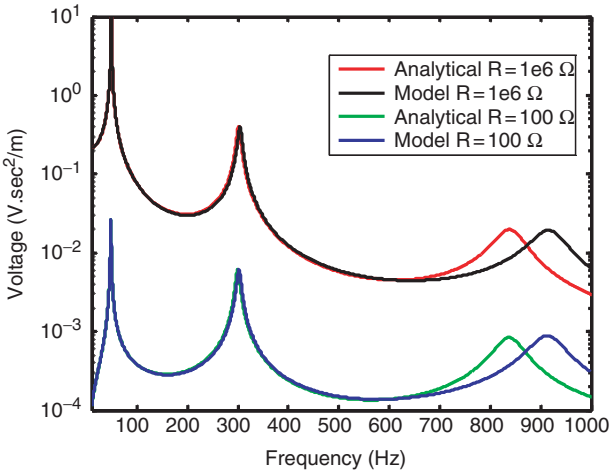

Figure 5. *Piezoelectric voltage versus frequency response. The red and green lines are the analytical solution of Erturk and Inman (2008); the black and blue lines are the equivalent circuit model solutions. The top set of curves corresponds to a circuit resistance of 1 MΩ, the lower curves for a resistance of 100 Ω.*

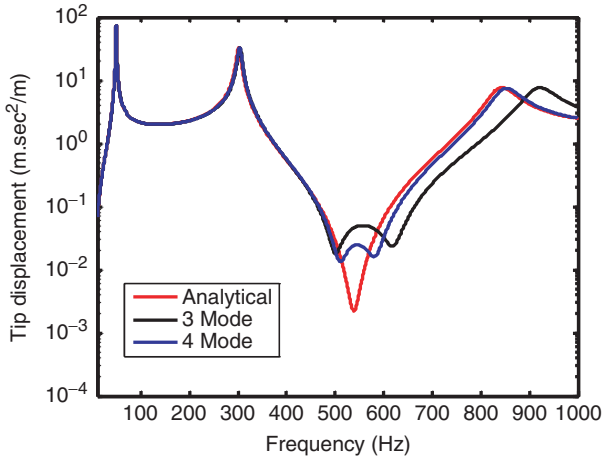

Figure 6. *Tip displacement versus frequency response. Red line is the analytical solution from Erturk and Inman (2007), back line is the solution from the equivalent circuit model considering three modes. For clarity the curve is only plotted for a resistance load of 1 MΩ. Blue line shows the tip displacement frequency response when the fourth mode is included into the equivalent circuit model.*

FURTHER EXAMPLE

The classical piezoelectric modeling approaches are difficult to use for analyzing more complicated and non linear electronic circuits attached to the piezoelectric harvester. The proposed equivalent circuit model allows for the analysis of these complicated circuits using standard circuit simulation tools. Consider a nonlinear circuit for converting ambient vibration to electrical charge on a storage capacitor (C_s) as shown in Figure 7. A leakage resistor (R_d) represents losses through the circuit.

The equivalent three-mode circuit is used to model the system in Figure 7. The piezoelectric properties are given in Table 1, with the storage capacitor $C_s = 3.3\,\mu F$ and

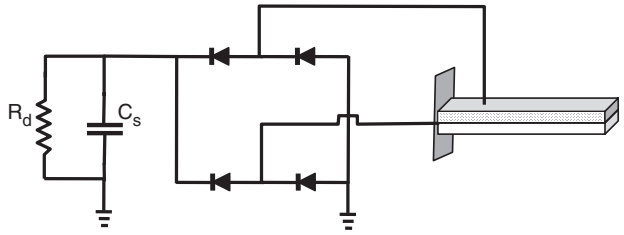

Figure 7. *A full diode bridge piezoelectric circuit for storing electrical charge on a capacitor (C_s). The circuit leakage is represented by the resistor (R_d).*

the leakage resistor $R_d = 10\,M\Omega$. The SPICE electrical circuit simulator is used to analyze the equivalent electromechanical and circuit systems, and to compute the average power that can be extracted from the storage capacitor. The average power is compared with the power that can be generated using only a resistor ($R = 100\,k\Omega$) as the circuit in Figure 1.

The average electrical power on the storage capacitor is less than for a purely resistive load over the entire frequency spectrum (Figure 8). The average power on the storage capacitor is calculated by assuming that the charge on the capacitor is completely drained every second i.e., the average power (P_a) is given by $P_a = C_s V^2/(2dt)$ where V is the voltage stored on the capacitor and dt is the charging period (i.e., $dt = 1$ sec). In this calculation, it is assumed that the capacitor is discharged instantaneously (i.e., the storage capacitor is assumed to be short circuited each second). The discharge of the capacitor would also be nonlinear and the actual power delivered to any realistic circuit would be nonlinear. The point of this simulation is to show that the proposed model can be used to accurately simulate complicated, nonlinear circuits. It is beyond the scope of this article to discuss optimal powerharvesting for full-bridge storage circuits. The power through the purely resistive circuit is given by $P_r = V^2/2R$.

For the unimorph operating at the first resonant frequency, the capacitor storage circuit generates only 40% of the power of an optimal resistive circuit.

The resistance circuit with $R = 100\,K\Omega$ and with the material properties given in Table 1 gives close to optimal power output at the first mode of the generator (Erturk and Inman, 2008). The resistor and capacitor in the storage capacitor circuit was chosen to give close to optimal power output at the first resonant mode of the generator. Figure 9 shows the average power output from the storage capacitor circuit as a function of the size of the storage capacitor C_s. The optimal power transfer from the storage capacitor shows similar behavior to a purely resistive load in the sense that the size of the optimal storage capacitor is also a function of base excitation frequency. This relatively simple example shows the ease with which complex circuits can be designed and optimized using the proposed equivalent circuit approach.

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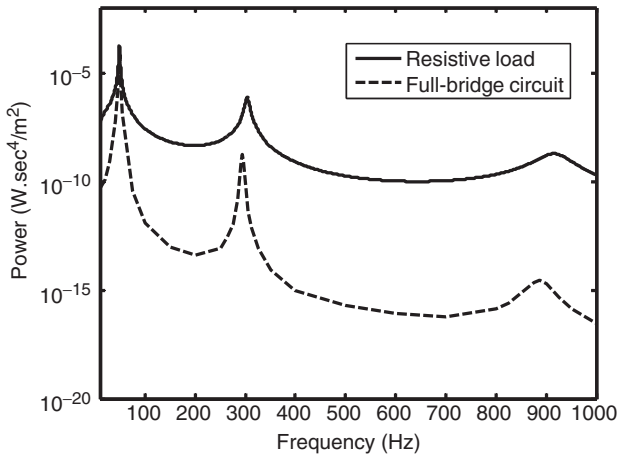

Figure 8. *The power frequency response of a purely resistive load (solid) and for a storage capacitor circuit (dashed). The resistance is 100 KΩ for the resistor circuit and $C_s = 3.3$ μF and $R_d = 10$ MΩ for the storage capacitor circuit.*

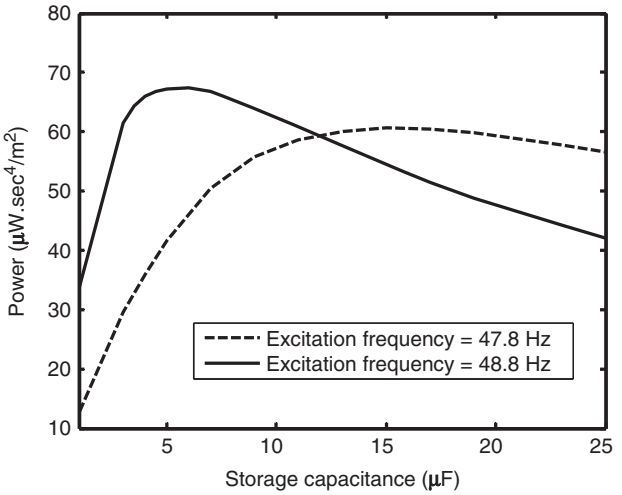

Figure 9. *The average power output from the full-bridge storage circuit for various storage capacitances (C_s) with a base excitation applied to the cantilever unimorph at the short circuit resonant frequency (47.8 Hz) and the open circuit resonant frequency (48.8 Hz). The short circuit resonant frequency response is shown by the dashed line, and the open circuit resonant frequency response is shown by the solid line.*

4. CONCLUSIONS

We have presented a novel equivalent circuit model for a piezoelectric energy harvester that can take into account higher vibrational modes. The equivalent circuit has been derived for a cantilever beam generator, but the method can be applied to any generator geometry and can include any number of vibrational modes. The advantage of the equivalent circuit is that it allows for the simple and efficient analysis of the coupled piezoelectric and circuits problems using a standard electrical simulation program such as SPICE.

The method consists of formulating the coupled electromechanical equations of motion using a standard Rayleigh–Ritz approach (though other methods such as the finite element method could also be used). The resulting mechanical equations are decoupled by solving the standard eigenvalue problem. The decoupled mass, damping, and stiffness equations can be readily replaced by an equivalent inductive, resistive, and capacitive circuit loop. The electromechanical coupling terms are then added to the circuit using an ideal transformer. It should be noted that the damping is assumed to be proportional which allows it to be decoupled (i.e., diagonalized). If other more complicated damping is assumed, the equivalent circuit could be coupled and thus more complicated to formulate.

As in all Rayleigh–Ritz approaches, the accuracy of the solution is dependent on the assumed mode shapes, and the number of assumed modes. It is recommended that the convergence of the solution should be tested by including more modes in the solution until an adequate accuracy is achieved.

The model has been verified against the analytical solution of Erturk and Inman (2008) for a resistive load unimorph cantilever generator that includes the first three bending modes. The analysis of a more complicated charge storage circuit has also been presented. The model was used to optimize the storage capacitive element under harmonic base excitation. The same approach could be used to optimize other piezoelectric generators and more complex circuits.

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